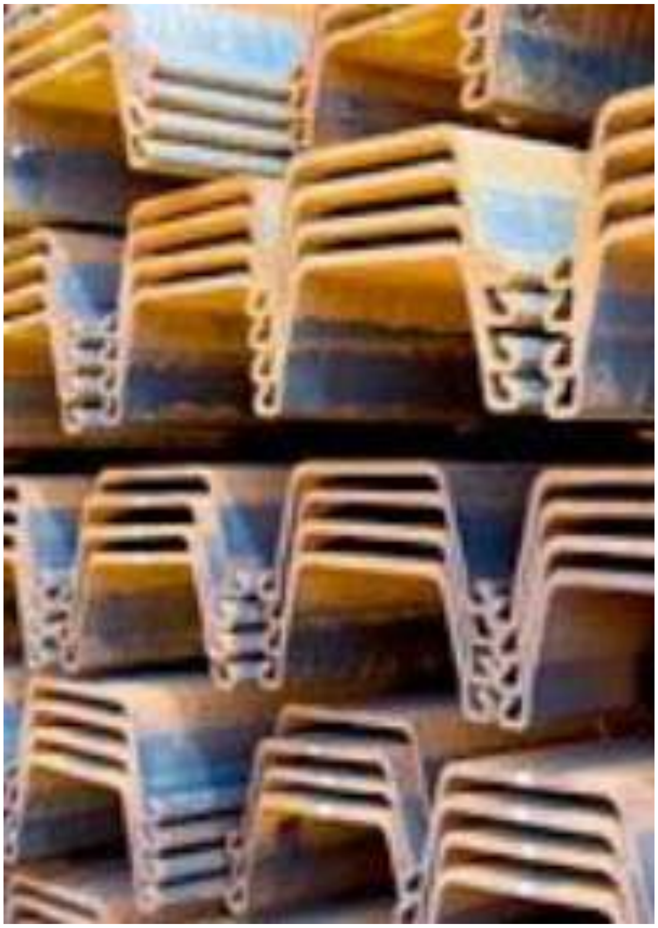

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Vibratory driving of piles and sheet piles – state of practice

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The installation of piles by vibratory methods is discussed and illustrated by case histories. The influence of vibration frequency – which can be controlled during vibratory driving but not during impact driving – on pile penetration, bearing capacity and emission of vibration is examined. The driving process affects the performance of vibratory-driven piles more strongly than impact-driven piles. Concepts are presented for assessing driveability. The importance of resonance of the vibrator–pile–soil system (system resonance) for driveability and pile bearing capacity is explained. Ground vibrations measured on and below the ground surface show that strong oscillating horizontal stresses are generated. These stresses can temporarily reduce the shaft resistance during driving, which can explain why vibratory driving is effective even in dense granular soils. Model tests show that, in granular soils, the bearing capacity of piles vibrated after driving at system resonance is significantly higher than that of piles installed only at high frequency. A concept is described suggesting that, during vibratory driving at high frequency, a zone is created adjacent to the pile shaft where the normal effective stress acting against the pile shaft is reduced due to arching. This arching effect can explain the reduced shaft resistance of piles installed at high frequency.

Notation

C_u	uniformity coefficient
c	wave propagation speed in pile
D_r	relative density
d_{10}	particles with diameter <10%
d_{50}	particles with diameter <50%
F_p	dynamic force in pile
F_v	centrifugal force
f	vibration frequency
I_D	density index (formerly relative density, D_r)
K_0	coefficient of lateral earth stress
L	pile length
M_e	eccentric moment
m_{cl}	mass of clamp
m_d	total dynamic mass
m_p	mass of pile
m_v	dynamic mass of vibrator
N_{10}	penetration resistance (number of blows per 0.1 m penetration)
S	peak-to-peak displacement amplitude
s	displacement amplitude
t_r	force transfer ratio
v	vibration velocity, particle velocity
v_P	pile penetration speed
z_s	specific soil impedance
Δs_i	change in displacement amplitude
$\Delta\sigma_h$	horizontally oscillating stress
λ	wavelength
ρ	total soil density
ϕ	friction angle
ϖ	circular frequency ($2\pi f$)

1. Introduction

In the foundation industry, heavy construction vibrators are primarily used to install sheet piles, whereas foundation piles are mostly installed by impact driving. However, the noise generated by impact driving can be a severe environmental issue in urbanised areas. Moreover, experience suggests that, in granular soil deposits, vibratory driving is the more efficient installation method (GDG, 2015; Holeyman, 2002; Rodger and Littlejohn, 1980; Viking, 2002; Warrington, 1992). However, in some cases, excessive ground vibrations have been reported as a result of vibratory driving (Athanasopoulos and

Pelekis, 2000; Deckner, 2013; Meijers and van Tol, 2005; Wiss, 1980).

The major reasons for the use of impact driving are that (*a*) reliable methods, such as wave equation analyses, can be used to predict the driving resistance for selecting suitable equipment and (*b*) after driving, stress wave measurements can be used to estimate the bearing capacity (Goble, 1994; Goble and Hussein, 1994; Goble *et al.*, 1975). In contrast, no reliable scientific tools are available for selecting vibratory driving equipment and assessing the bearing capacity of piles after installation (Bosscher *et al.*, 1998; O'Neill *et al.*, 1990; Rausche, 2002; Schönit, 2009). As will be discussed in subsequent sections, there is uncertainty with regard to the bearing capacity of vibrated piles. Nevertheless, vibratory driving of piles has recently become more frequently used, especially for the installation of large tubular piles in the offshore industry (Foglia *et al.*, 2016; Jonker, 1987; Lamens *et al.*, 2020; Matuschek and Betke, 2009; Remspecher, 2014). That experience has shown that, under suitable ground conditions and with the use of powerful equipment, vibratory driving can be faster and more cost effective than impact driving. However, it is general practice to improve the bearing capacity of piles by subsequent impact driving, often coupled with dynamic testing.

Another area of foundation engineering where heavy construction vibrators are used more extensively is deep vertical vibratory compaction (DVVC). Different types of purpose-built compaction probes have been developed, which are inserted in the ground using a vertically oscillating vibrator (Anderson, 1974; Li *et al.*, 2018; Liu and Cheng, 2012; Massarsch, 1991; Massarsch and Broms, 1983; Massarsch and Fellenius, 2019; Mitchell, 1981). After full penetration, the probe is extracted and reinserted in steps. A special development of DVVC is resonance compaction – here, the vibration frequency is varied during penetration and subsequent extraction. During the compaction phase, the probe is operated at the resonance frequency of the vibrator–probe–soil system (subsequently called the system resonance frequency), thereby amplifying ground vibrations and thus the compaction efficiency (Massarsch and Fellenius, 2017; Massarsch and Heppel, 1991). Vibration sensors (geophones) on the ground surface can be used to determine resonance effects by measuring the variation of vertical vibration velocity as a function of frequency. Recently, a concept was proposed to use the resonance compaction technique for full-scale testing of soils that could be susceptible to liquefaction (Massarsch *et al.*, 2019a, 2019b).

This paper summarises the experience of vibratory driving over the past 40 years, with emphasis on its practical application. In addition to an extensive literature review covering experiences from mainly Europe and North America, previously unpublished and published data have been re-evaluated and put into a general context, shedding new light on the vibratory driving process and its effect on pile bearing capacity. For the efficient and environmentally friendly use of vibratory driving, the following aspects must be considered by geotechnical engineers

- driveability: selection of adequate vibrator capacity and choice of execution parameters (frequency, eccentric moment)
- environmental impact: emission of vibrations and noise to the surroundings, settlement, soil movement and potential impact on the stability of slopes and excavations
- bearing capacity: assessment of the bearing capacity and stiffness of the pile.

In the following text, unless specifically mentioned, the term pile refers to preformed piles, sheet piles or compaction probes.

2. Difference between impact and vibratory driving

There are fundamental differences between the process of impact driving and vibratory driving, which are not generally appreciated. The differences between impact and vibratory pile driving have been discussed by, for example, Dierssen (1994) and GDG (2015). The installation method can have a significant effect on the pile penetration resistance, environmental impact and pile bearing capacity. These aspects are discussed in the following sections.

2.1 Impact driving

The process of impact pile driving has been described in detail in the literature (e.g. Broms and Bredenberg, 1982; Goble, 1994). A less frequently addressed aspect of impact pile driving is the propagation of vibrations from the pile to the surrounding soil and the dynamic response of soil layers (Massarsch and Fellenius, 2008). Figure 1 illustrates the vibrations generated due to impact pile driving. A hammer blow is transmitted to the pile head (through a cushion and pile cap), generating a short-duration stress wave. The hammer is in contact with the pile only for a very short time. The stress wave propagates to the toe of the pile and is there reflected back up towards the pile head, which is when the hammer–pile contact is broken. In the process, the driving energy is transferred to the surrounding soil along the shaft as well as at the toe. Next to the pile, a short-duration, rapidly decaying impulse is generated outward into the soil. An important but often disregarded factor is the frequency spectrum of the vibration time history, which shows peaks that correspond to resonance frequencies of the soil deposit (Massarsch and Westerberg, 1995). However, the frequency response of the ground vibrations created by the pile impact cannot be controlled.

2.2 Vibratory driving

In the case of vibratory driving, counter-rotating eccentric masses generate a centrifugal force. The magnitude of the

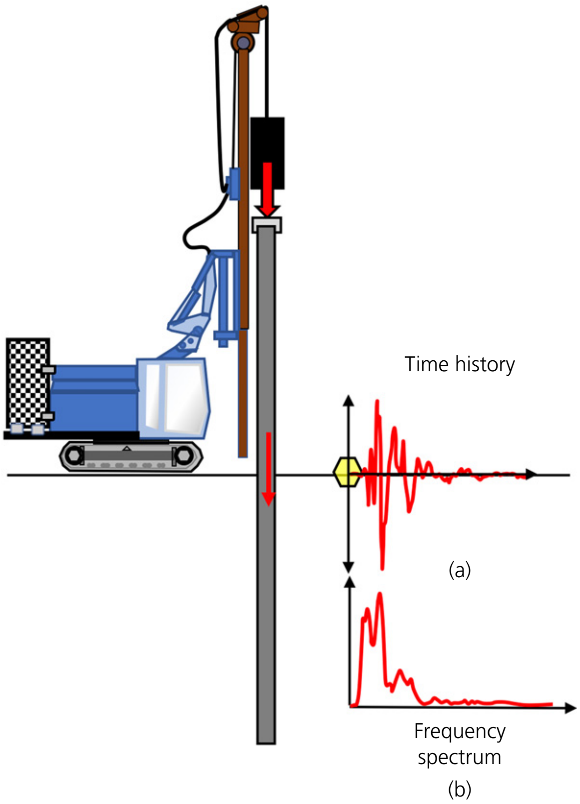

Figure 1. Illustration of impact driving of pile and vibration propagation into the surroundings, showing velocity time history (a) and frequency spectrum (b) of vibrations at the ground surface

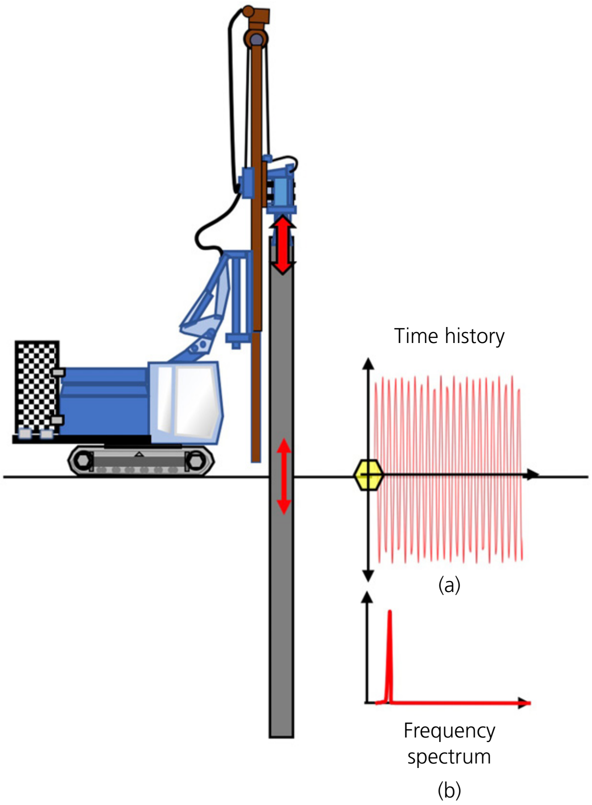

Figure 2. Illustration of vibratory driving of pile and generated ground vibrations: (a) time history; (b) frequency spectrum. Note that the vibrator is rigidly connected to the pile

centrifugal force depends on the eccentric moment and the frequency of the rotating masses. Vibratory driving of a pile is illustrated by Figure 2. The vibrator is rigidly connected to the pile and causes the pile to oscillate in the vertical direction. The rigid connection prevents energy loss occurring between the vibrator and the pile. However, the machine operator can also control pile penetration by applying a tension force (pulling the vibrator) – often unintentionally. This effect is especially important during the final driving phase of pile installation. The vibratory driving process will be discussed in detail in subsequent sections.

As the loading frequency is low (typically 20–40 Hz) compared with impact pile driving, the length of the stress wave in the pile exceeds the length of the pile, which then moves essentially as a rigid body and therefore stress wave propagation effects in the pile can be neglected. However, when vibratory driving occurs onto a hard layer or rock (impacting of the toe against an unyielding base), stochastic pile movements and superimposed impact vibrations can occur (Sieffert, 1980, 1984). Rodger and Littlejohn (1980), Holeyman (2000), Viking (2002) and Deckner (2017) have discussed the transfer of energy to surrounding soil during vibratory driving. The effect of vibration frequency on the execution of vibratory pile driving has been considered in only a few cases (Hartung, 1994; Massarsch, 1992; Massarsch *et al.*, 2017).

Vibrations are transmitted from the shaft and the toe of the pile to the surrounding soil. The number of vibration cycles is significantly larger during vibratory driving than during impact driving. The most important aspects are (*a*) during each vibration cycle, the toe of the pile lifts off from the underlying soil and (*b*) the interaction between the pile shaft and the surrounding soil is governed by the frequency of the vibrating system with the critical parameter of vibratory driving being the resonance frequency of the vibrator–pile–soil system (Massarsch *et al.*, 2017). This aspect will be discussed in detail in subsequent sections.

Ground vibrations are sinusoidal with a distinct peak in the frequency spectrum (see Figure 2). The time history shows sinusoidal vibrations and the frequency spectrum has typically only one peak, which corresponds to the operating frequency of the vibrator. The spectrum may also include overtones of the operating frequency. If the operating frequency of the vibrator approaches the system resonance frequency, the ground vibrations will be strongly amplified. As the vibration frequency can be adjusted during vibratory driving, resonance effects can be controlled (avoided or amplified). Also, the vibration amplitude of the pile can be adjusted by changing the eccentric moment (see Section 3). In contrast to impact driving, it is possible to control the driving process with modern vibrators by adjusting the eccentric moment and the vibration frequency (and thus the centrifugal force).

Soil vibrations attenuate with increasing distance from the source (the pile) and at increasing frequency, but the dominant frequency (the driving frequency) remains essentially unchanged (Figure 2).

In the case of vibratory pile driving, three types of resonance can occur: (*a*) resonance of a soil layer resting on a rigid base (soil layer resonance); (*b*) resonance of a pile element (pile resonance) and (*c*) resonance of the vibrator–pile–soil (system resonance). In the following sections, the significance of system resonance, which is important for vibratory driving and pile bearing capacity, will be discussed.

2.3 Wavelength

The vibrations induced by impact driving (impact stress wave) and vibratory driving (sinusoidal vibration) are fundamentally different. The wavelength, λ, of a sinusoidal wave can be estimated from

$$1. \quad \lambda = \frac{c}{f}$$

where c is the wave speed in the pile and f is the vibration frequency.

When the wavelength of the stress wave is significantly longer than the length of the pile, the pile will respond as a rigid body. The wave speed is about 3500 m/s in a concrete pile and about 5000 m/s in a steel pile. For a vibration frequency of, say, 30 Hz, the wavelength is in the range 100–150 m, thus significantly longer than that of the pile (Massarsch, 2000). The differences between vibratory and impact driving are summarised in Table 1.

2.4 Other factors affecting vibratory driving

The sheet pile geometry and the elastic section modulus can affect driving due to longitudinal, transversal and flexural excitations (Deckner, 2017; Osthoff, 2017; Viking, 2002). The efficiency of vibratory driving is also influenced by several other factors, such as the verticality of installed piles or the eccentric positioning of the vibrator with respect to the neutral axis of the sheet pile. While the eccentric application of the centrifugal force can cause flexing of the sheet pile, especially above the ground, no quantitative information has yet been published that shows whether – and to what extent – this can affect the driving process. Another important factor is the interaction during driving of interlocking sheet piles. As sheet piles are frequently reused, their condition, damage to locks, shape and straightness can vary. When sheet piles are installed as part of a wall, the resistance to driving can be affected if adjacent sheet piles are bent or inclined (Vanden Berghe *et al.*, 2001; Viking, 2002). Such effects are difficult to predict and need to be studied on a project-specific basis, preferably by field trials.

3. Fundamentals of vibrator operation

3.1 Vibrator types

Electric vibrators were first developed in the mid 1950s in the Soviet Union and were subsequently introduced in Japan and Europe (Warrington, 1992). Hydraulic vibrators were developed in Europe in the mid 1960s and are now widely used in Europe and North America. Vibrators for pile driving typically operate in a frequency range of 10–40 Hz. Detailed descriptions of different types of vibrators and their operation have been presented by, for example, Warrington (1992), Massarsch (2000) and Viking (2002).

In the 1960s, vibrators operating at very high frequencies (>100 Hz) were developed in North America. The objective was to drive piles at the resonance frequency of the pile, which is much higher than the system resonance frequency. The so-called 'Bodine resonant pile hammer', developed by Bodine and Guild, was used for pile driving but suffered from operational problems (Warrington, 1992). Recently, a new type of high-frequency vibrator (resonant driver) was introduced,

Table 1. Comparison of impact and vibratory driving

Action	Impact driving	Vibratory driving
Hammer–pile connection	No: hammer and pile separated	Yes: hammer clamped to pile
Excitation	Short-duration impact	Steady-state motion
Driving force	Impulse	Sinusoidal
Length of stress wave in pile	Short ($<0.1L$)	Long ($>10L$)
Number of vibration cycles	Small	Large
Pile excitation	Stress wave	Rigid-body movement
Driving energy depends on	Hammer mass, drop height, energy loss in pile cap/cushion	Centrifugal force, vibration frequency, eccentric moment
Pile shaft–soil interaction	Cannot be controlled	Controlled by vibration frequency
Pile toe in contact with soil	Yes	Intermittently
Ground response	Resonance generated at each blow	Ground response and resonance can be controlled by driving frequency

which uses a hydraulic piston–cylinder to generate vibrations at very high frequencies (up to 180 Hz). High production rates and low ground vibrations have been reported from sheet pile installation projects (Janes, 2014). However, one important aspect is that the resonance frequency of the pile changes (decreases) when the pile toe penetrates into hard layers, which means that maintaining the desired resonance frequency of the pile can become more difficult.

3.2 Vibrator characteristics

Modern vibrators are sophisticated machines and their performance can be adapted to achieve efficient driving with minimum environmental impact. However, vibratory driving can also potentially become slow and cause excessive vibrations when the vibrator is operated close to the system resonance frequency. The vertical oscillation of the vibrator is generated by counter-rotating eccentric masses. The operating frequency of modern vibrators can be varied during driving without reducing the hydraulic power. The centrifugal force, F_v, depends on the eccentric moment, M_e and the circular frequency, ϖ, of the counter-rotating eccentric masses

$$2.\quad F_v = M_e\omega^2$$

Another important factor in vibrator performance is the displacement amplitude, s (the integral of the vibration velocity). The displacement (without regard for dynamic effects) can be calculated from

$$3.\quad s = \frac{M_e}{m_d}$$

where m_d is the total dynamic mass of the vibrating system. The total dynamic mass is the sum of all masses that must be accelerated by the vibrator – the dynamic mass of the vibrator, m_v, the mass of the clamp, m_{cl} and the mass of the pile, m_p. Thus

$$4.\quad m_d = m_v + m_{cl} + m_p$$

Equipment manufacturers usually state the peak-to-peak (double) displacement amplitude, S (2s). The vibration amplitude can be of importance for the driving efficiency and should be checked in order to choose a vibrator with sufficiently high eccentric moment. If, for instance, $M_e = 25$ kg.m, $m_v = 2500$ kg and $m_{cl} = 500$ kg, then, according to Equations 3 and 4, the nominal peak-to-peak displacement amplitude, S, is 16.7 mm. However, when the mass of the pile is added ($m_p = 1000$ kg), S decreases by 25% to 12.5 mm. From Equation 3, it can be seen that the displacement amplitude is independent of the vibration frequency, f.

The centrifugal force, F_v, generated by a vibrator with dynamic mass m_v is transferred to the pile with mass m_p. Whenham and Holeyman (2012) showed that if the pile is assumed to behave as a rigid body (constant acceleration throughout the pile), the force applied to the pile head, F_p, is an internal force within the vibrating system. Thus, the dynamic force in the pile F_p is smaller than the centrifugal force F_v, as the vibrator must also accelerate the masses of the pile and the vibrator clamp (m_p and m_{cl}). According to Whenham and Holeyman (2012), the forces and masses of the vibrating system can be described by

$$5.\quad \frac{F_v}{m_v + m_{cl} + m_p} = \frac{F_v - F_p}{m_v} = \frac{F_p}{m_p + m_{cl}}$$

It is possible to define a force transfer ratio, t_r (i.e. the difference of the force acting on the head of the pile F_p in relation to the centrifugal force F_v):

$$6.\quad t_r = \frac{F_v - F_p}{F_v} = \frac{m_v}{m_v + m_{cl} + m_p}$$

Once the mass of the pile and clamp are known, the internal force actually applied to the head of the pile can be calculated using Equation 6. If, for instance, the mass of the pile and clamp is equal to the dynamic mass of the vibrator, then $t_r = 0.5$. Thus, only 50% of the centrifugal force is actually transmitted from the vibrator to the head of the pile. It should be noted that this effect of added mass also reduces the displacement amplitude, as discussed earlier.

In order to increase the driving performance of vibrators, an external static mass can be mounted on top of the vibrator, supported by elastomeric springs. On some piling rigs it is also possible to use the piling rig to apply an additional vertical pushing force. Similarly, due to the rigid connection between the vibrator and the pile, a tension force can be applied, which can have a significant effect on pile bearing capacity, as will be discussed in later in the paper.

4. Pile penetration resistance during vibratory driving

4.1 Vibratory driving resistance

The penetration resistance during vibratory driving of piles (assuming the rigid-body concept) is generated simultaneously along the pile shaft and at the pile toe. Different concepts have been proposed to explain the efficiency of vibratory driving in granular soils. Kühn (1978) introduced the term ‘pseudo-fluidisation’ (*pseudoflüssiger Zustand* in German) to explain the efficiency of vibratory driving in granular soils. This concept has, by some authors, been incorrectly termed and interpreted as ‘liquefaction’ (*Bodenverflüssigung*). Another frequently mentioned phenomenon is that, during vibratory penetration, a ‘rolling friction’ causes a reduced shaft friction (Warrington, 1992). A common opinion expressed is that during vibratory driving in granular soils, the soil strength immediately adjacent to the shaft (the shaft resistance) is effectively reduced due to cyclic displacement (Fleming *et al.*, 2008). It is generally accepted that the centrifugal force is the main factor influencing vibratory pile driving in granular soils, while the displacement amplitude is a critical parameter in cohesive (fine-grained) soils.

Different factors influencing pile penetration during vibratory driving have been discussed extensively in the literature (Dierssen, 1994; Holeyman, 2002; Sieffert, 1984; Viking, 2002). Analytical models have been developed for the assessment of pile resistance during vibratory driving. Holeyman (2000) described a procedure for modelling the dynamic non-linear pile–soil interaction and also critically reviewed the ability to assess the capacity of a vibratory-driven pile from monitoring vibrations. Rausche (2002) proposed a model for estimating the driving resistance based on stress wave theory and found reasonable agreement between predictions and analysis of stress wave measurements. An overview of different methods of calculating the driving resistance was presented by Whenham and Holeyman (2010).

Rodger and Littlejohn (1980) presented a comprehensive driveability study of model piles in dry, granular soil. A model vibratory driver (0.745 kW) was used to drive an instrumented 38 mm dia. closed-toe steel pile into a bed of dense fine/medium-grained sand ($C_\text{u} = 1.2$, $d_{10} = 0.29$ mm, $D_\text{r} = 71.5\%$, $\phi = 41°$). The study distinguished three different penetration modes

(*a*) slow vibratory driving, where reversal of motion occurs, during which the soil is unloaded completely
(*b*) fast vibratory driving in which reversal of motion occurs but, because some contact stress remains at the pile toe, the toe resistance remains
(*c*) pulsating vibratory driving in which there is no reversal of motion, the pile bounces and motion is unsystematic.

Slow vibratory driving (*a*) is considered to be the most widely encountered case and is the focus of the following discussion. At first, the pile toe resistance will be examined, followed by a discussion of the pile shaft–soil interaction.

4.2 Pile toe resistance

The mechanism of pile toe–soil interaction during slow vibratory driving has been studied by several investigators (Cudmani, 2001; Deckner, 2017; Holeyman, 2002; Viking, 2002). Figure 3 shows a simplified model of the motion of the pile toe during one vibration cycle (Dierssen, 1994; Tseitlin *et al.*, 1987). The figure shows the variation of the toe force, F_v, as function of pile penetration (i.e. displacement amplitude, s) and illustrates the driving process at the pile toe during slow vibratory driving. The pile motion and interaction between the pile toe and the underlying soil is now addressed (the effect of shaft friction is omitted for clarity).

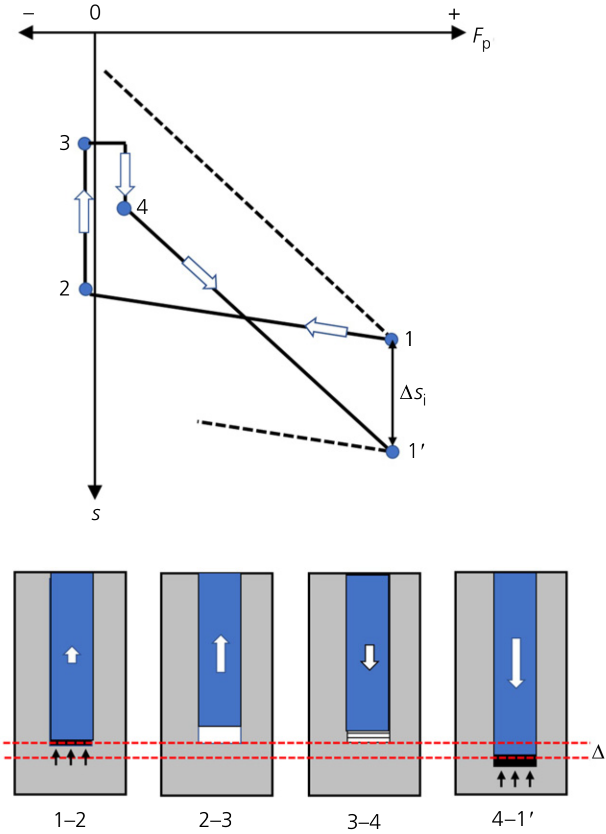

Figure 3. Simplified model of rigid pile toe–soil interaction during vibratory driving (slow vibratory driving). The diagram shows force in pile F_p against pile toe displacement Δs_i (modified from Dierssen (1994))

At point 1 in Figure 3, the pile has completed the downward motion and starts the upward rebound movement of a vibration cycle. At first, the soil below the pile toe will follow the upward movement in an elastic response. At point 2, the contact force between the pile toe and the soil has reduced to zero. If the upward movement of the pile stops at point 2, the pile toe will not become separated from the underlying soil. This will be the case if the displacement amplitude of the pile is small. If, on the other hand, upward movement of the pile continues, the pile toe separates from the underlying soil. During this state, a void can be created between the soil and the pile toe, causing decompression and, in water-saturated soils, potentially suction (cavitation). This phase of vibratory driving is important for the penetration process as it can result in remoulding and/or loosening of the soil below the pile toe. If there is little or no separation between the pile toe and the underlying soil, the resistance to downward movement (penetration) will be approximately similar to that generated by impact driving. It is apparent that the vibration amplitude influences the toe/soil resistance during vibratory driving. The mode of toe penetration during vibratory driving also affects the toe bearing. If the vibration amplitude – and therefore the toe–soil separation (points 2–3) – is small during the final phase of pile installation, soil decompression below the toe can be avoided, resulting in enhanced toe resistance. When the oscillation cycle is reversed (point 3), the pile again moves downward and, after some movement, regains contact with the soil (point 4). The toe resistance during the following cycle (point 1′) depends on the effect the previous vibration cycle had on the soil below the toe. For this reason, it is advantageous to reduce the displacement amplitude gradually during the termination of driving toe-bearing piles. If the machine operator unintentionally pulls (or holds back) the pile during the final phase of driving, this may have detrimental effects on the soil stiffness below the pile toe. This is an important aspect that is not generally appreciated and can explain the low toe resistance measured by pile loading tests.

Many challenges remain when estimating the toe resistance of vibratory-driven piles and sheet piles. One unresolved issue is the influence of the toe geometry, which differs between sheet piles, open-toe pipe piles and closed-toe pipe piles. For instance, the formation of a soil plug in a pipe pile is still an unresolved issue. The vibration frequency (resonance effects) can influence the formation of the plug at the toe of a pipe pile.

4.3 Shaft resistance

In the case of vibratory driving, the pile shaft is kept oscillating during the entire driving process. Several hypotheses have been offered to explain why, in coarse-grained soil, the shaft resistance during vibratory driving is significantly lower than during impact driving. Different reasons for the reduced shaft friction have been proposed, such as reduced rolling friction, liquefaction, fluidisation, friction fatigue and material degradation. Different concepts have also been developed for analysing the shaft resistance (Cudmani, 2001; Dierssen, 1994; Holeyman, 2000; Massarsch, 2002; Moriyasu *et al*., 2018; Rausche, 2002). Viking (2002) discussed the effect of volume change during pile penetration in terms of ‘contractive and dilative behaviour of the soil’.

DeJong *et al*. (2003) studied the effect of cyclic (wave) loading on the shaft resistance of piles in sand. Monotonic and cyclic interface shear tests were performed using a modified interface direct shear device. The results indicated that the confinement condition, which is intended to model the elastic response of far-field soil, is of primary importance as it allows for normal stress relaxation with soil contraction adjacent to the interface. The displacement magnitude, particle characteristics and particle–particle cementation were also observed to affect the magnitude and rate of degradation. The cyclic degradation of shear stress arises due to cumulative contraction of the shear band close to the interface. It was noted that the magnitude of the contraction increases with an increasing number of cycles.

Lehane and White (2005) described a series of tests performed in a drum centrifuge on instrumented model displacement piles in normally consolidated sand. The piles were installed using different methods (monotonic, jacked and pseudo-dynamic). Although the cycling associated with pile installation resulted in a progressive reduction in the static horizontal effective stress acting on the pile shaft and densification of the sand in a shear band close to the pile shaft, this sand dilated strongly during subsequent shearing to failure in a static loading test. The dilation (the amount of which depends on the cyclic history) is constrained by the surrounding soil and therefore leads to large increases in lateral effective stresses and hence to large increases in mobilised shaft friction. The lateral stress changes that take place during pile loading have a dominant influence on the shaft friction that can develop on small-scale piles in sand. However, the tests did not simulate the cyclic installation process that occurs during vibratory driving.

Vogelsang *et al*. (2017) reported an experimental study using model tests to investigate the vibro-penetration of piles in saturated sand. In the tests, a model pile with a half-circular cross-section was installed along an observation window by means of a vibrator. A high-speed camera and a sophisticated image acquisition system were used to observe the penetration process. The investigation showed the typical displacement patterns in the soil during cyclic pile penetration. Vogelsang *et al*. (2017) described pore water pressure measurements at two fixed locations, which showed the dependence of pore water pressure evolution on the penetration mode.

5. Ground vibrations due to vibratory driving

In order to gain a better understanding of the effects of vibratory driving on the surrounding soil, vibration measurements on – or in – the ground can provide valuable information. Usually, vibration measurements are performed on the ground surface or in buildings in order to monitor the effect of vibratory driving (Clough and Chameau, 1980). Another objective of such measurements is to assess the risk of settlements due to ground vibrations (Massarsch, 2000; Meijers and van Tol, 2005; O’Neill, 1971). The effect of vibratory driving on the stability of slopes has also been investigated (Lamens *et al*., 2020). In all these studies, vibration sensors (usually geophones) were installed at the ground surface and the vibration velocity was measured at different distances from the vibration source.

5.1 Impact driving

Only a few vibration measurements have been reported where the sensors were installed below the ground surface. Woods *et al*. (2014) reported vibration measurements during impact driving of H-piles in loose sand. Vibration sensors were placed at three depths very close to the pile shaft (within 0.15 m) to measure the resulting ground motion during impact pile driving. The data suggested that when the pile toe is far above the sensor, vibrations are emitted mainly from the pile toe. As the toe gets closer to the sensor level, a contribution from the pile shaft also reaches the sensor. After the pile toe has passed the depth of the sensor, the larger vibrations from the pile toe and shaft combine. The vibration levels after the toe passes the sensor elevation stay relative constant, suggesting that the pile toe – when moving further from the sensor – contributes less and less to the vibration than the shaft in the zone nearest the pile.

5.2 Vibratory driving

Results from vibration measurements during vibratory driving and vibratory compaction in sand have been reported by Krogh and Lindgren (1997). The purpose of these measurements was to investigate the change in horizontal stress resulting from DVVC. Several case histories have been reported where a permanent increase in horizontal effective stress was measured by cone penetration tests (CPTs) and flat dilatometer tests (e.g. Massarsch, 1991; Massarsch and Fellenius, 2014; Massarsch *et al.*, 2019b). In order to investigate the source of the horizontal stress increase, horizontal geophones were installed at a lateral distance of 2.9 m from the centre of the compaction probe, at the ground surface and at three depths (1.65, 3.55 and 5.05 m). During initial probe penetration, the compaction probe was vibrated with a frequency of 24 Hz. An example of horizontal vibration velocity during penetration of the probe at 2.9 m distance is shown Figure 4 (the probe depth was 6.0 m).

Horizontal vibrations were emitted from the vertically oscillating probe and the vibration amplitudes were almost constant with depth. The horizontal ground vibrations were sinusoidal and the vibration frequency was equal to the operating frequency of the vibrator. It can be seen that strong horizontal stress pulses were directed away from – as well as towards – the compaction probe. As a result of a large number of vibration cycles, soil will be subjected to strong horizontal stresses variations. The horizontal vibrations shown in Figure 4 are the result of compression waves emitted from the oscillating probe. The horizontal stress change, $\Delta\sigma_h$, can be estimated from the specific soil impedance, z_s, and the particle velocity, v (Massarsch and Fellenius, 2008):

$$7.\quad \Delta\sigma_h = vz_s = vc^*_P\rho$$

where c^*_P is the compression wave speed and ρ is the total soil density. If it is assumed that the horizontal vibration velocity v is 25 mm/s, the compression wave speed is 1450 m/s (corresponding to the P-wave speed of the groundwater) and the total soil density is 2000 kg/m^3, $\Delta\sigma_h = 72$ kPa. It should be noted that higher horizontal vibration velocities can occur during the compaction phase. A hypothesis to explain why vibratory driving is effective in granular soils was proposed by Massarsch (2002). This hypothesis will be discussed in sections 9 and 10 this paper.

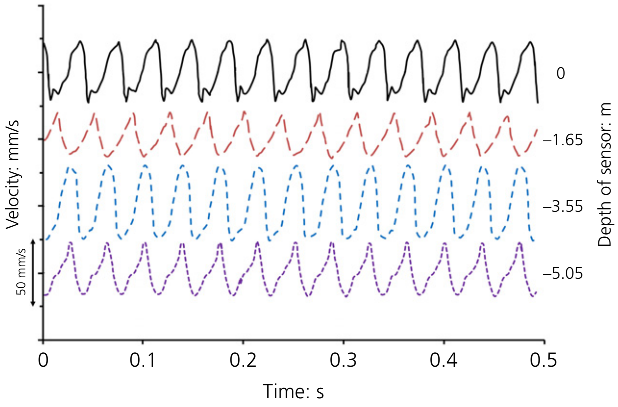

Figure 4. Horizontal ground vibrations measured at 2.9 m distance from vertically oscillating probe during the penetration phase. The depth of the toe was 6 m. The depth of sensors is indicated in the diagram (data from Krogh and Lindgren (1997))

6. Driveability assessment

An important part of project design is the selection of the optimal vibrator capacity (eccentric moment and centrifugal force) for the installation of piles or sheet piles. Equally important is to optimise the driving process (eccentric moment and vibration frequency). In addition, the pile type (sheet pile, closed- or open-toe pipe pile, compaction probe) and pile size (length and mass) affect the driveability. Three alternatives are available to project engineers when performing a driveability analysis: (*a*) empirical methods, (*b*) dynamic analyses and (*c*) field observations and back-analysis.

6.1 Empirical methods

Empirical methods have been developed by vibrator manufacturers to assist clients in their selection of suitable vibrators. Unfortunately, little factual evidence has been published regarding the scientific basis and limitations of these empirical correlations. As has been shown above, the driving resistance is affected by the type of pile (pipe pile with closed or open toe or sheet pile). An example of a driveability chart, presented by Massarsch (2000), is shown in Figure 5. The selection of a vibrator with sufficient centrifugal force (which depends on eccentric moment and vibration frequency) is based on the total mass of the pile and the penetration depth. The assessment of soil resistance is based on five soil categories. The approximate soil classification according to Figure 5 is given in Table 2. In Figure 5, the practical application of the chart is illustrated by an example. If it is assumed that a sheet pile of mass 4 t is driven to a depth of 20 m into a dense sand (category IV), the required centrifugal force of the vibrator is about 2000 kN.

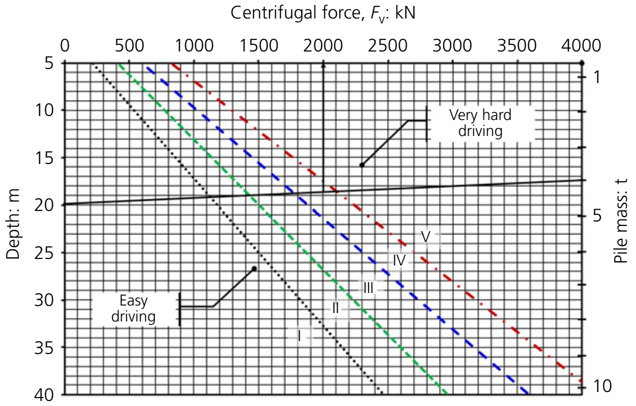

Figure 5. Required centrifugal force for pile installation in granular soil, based on pile mass and driving depth, based on Massarsch (2000). Soil classifications I–V are shown in Table 2

A more reliable method for estimating the vibratory driving resistance can be based on different types of penetration tests. If it is assumed that the vibrated pile acts as a rigid body, the driving resistance can be divided into two components: toe resistance and shaft resistance. Westerberg *et al.* (1995) developed a simplified approach of estimating the vibratory driving resistance, based on CPTs. They assumed that the toe resistance during vibratory driving can be correlated approximately to the results of the CPT (cone resistance and sleeve resistance). However, while the response of a vibrated pile during static penetration is approximately similar to that of a CPT, the shaft resistance will be significantly lower during vibratory driving at high frequency. It should be noted that the shaft resistance changes when the pile is vibrated at the system resonance frequency. At resonance, the pile oscillates in phase with the surrounding soil and the relative displacement between the shaft and the soil decreases.

6.2 Dynamic analyses

Different types of driveability analysis methods have been proposed, based on stress wave propagation. An early concept was the 'beta formula method', proposed by Jonker (1987) for predicting the driving resistance of pipe piles in the offshore industry. The objective of dynamic analyses is to estimate the dynamic penetration resistance as well as the penetration speed during vibratory driving. An overview of different methods is presented by Viking (2002). Theoretical modelling of vibratory driving of piles is possible (Holeyman, 2000; Rausche, 2002), but a major problem is the selection of realistic geotechnical properties representing the dynamic pile–soil interaction (Bosscher *et al.*, 1998). Viking (2002) compared different prediction models with field tests and concluded that the results of theoretical vibro-driving models differ from experimental evidence and can lead to significant errors in the driveability assessment. Whenham and Holeyman (2010) compared vibro-driving prediction methods with experimental data and results from full-scale sheet pile vibro-driving tests. Whenham (2011) provides a detailed discussion of the concepts of vibratory driving analyses. Holeyman and Whenham (2017) evaluated the application of the vibratory drivability of piles and sheet piles developed by (Holeyman, 1993), based on experimental results.

6.3 Field observations

Due to the limitations of theoretical concepts to predict driveability, an alternative approach is to use full-scale tests to compile site-specific correlations of driveability. Vibratory driving of piles can be envisaged as a full-scale dynamic penetration test, provided that the driving parameters (frequency, eccentric moment) are known and kept constant. In granular soil, the total vibratory driving resistance of a pile is dominated by the toe resistance, while the shaft resistance contribution is relatively small – but gradually increases with depth. A key parameter is the measurement of the pile penetration speed at a constant vibration frequency (Massarsch *et al.*, 2017). This concept was originally developed as part of the resonance compaction method to establish a correlation between the degree of required compaction (in terms of penetration resistance) and the probe penetration speed.

Schönit (2009) reported field tests where a 9.5 m long H-beam (Peiner PSp 370) was installed by a vibrator with variable frequency (MS-10 HFV) and an adjustable eccentric moment ($M_e = 0–10$ kg.m). The dynamic mass of the vibrator was 1700 kg. The maximum centrifugal force was 610 kN. The mass of the beam was 122 kg/m. Test piles (beams) were installed at three frequencies (25, 30 and 40 Hz). The field trials were carried out at the test field of the University of Karlsruhe, Germany. The ground consisted of a granular soil deposit of sand and gravel. The soil conditions are described in detail by Schönit (2009). The groundwater table was located 5.4 m below the ground surface. The penetration resistance was measured by a light dynamic penetrometer with a drop

Table 2. Approximate density classification of granular soils

Category	Density	CPT: MPa	Super heavy dynamic probing: blows/0.20 m	Standard penetration test: blows/0.3 m
I	Very loose	<2	<4	<4
II	Loose	2–4	4–8	4–10
III	Medium dense	4–12	8–12	10–30
IV	Dense	12–20	12–25	30–50
V	Very dense	>20	>25	>50

mass of 10 kg and a fall height of 0.5 m. The penetration resistance was measured as the number of blows per 0.1 m penetration (N_{10}). The average penetration resistance of 15 penetrometer tests is shown in Figure 6. The surface layer consisted of loose to medium-dense sand ($N_{10} = 10$–15) followed (between 4 and 5 m depth) by a gravel layer ($N_{10} = 20$–35). The underlying sand had a low penetration resistance, increasing from 6 m ($N_{10} \approx 7$) to 8 m ($N_{10} \approx 20$). The zone of interest for this investigation was between 3.5 m and 8 m.

The penetration speed (m/s) of the 9.5 m long H-beam was measured for a vibrator with an eccentric moment of 10 kg.m at three frequencies (25, 30 and 40 Hz). The measured speed distributions are shown in Figure 7. The penetration speed was generally high down to 4.5 m depth, reflecting the loose sand, but decreased in the gravel layer at 4.5–5 m depth. Below the gravel, the penetration speed increased again in the looser sand layer. The average penetration speed was approximately three to five times higher at 40 Hz than at the lowest frequency of 25 Hz. Unfortunately, the system resonance frequency is not known as no system resonance tests were performed.

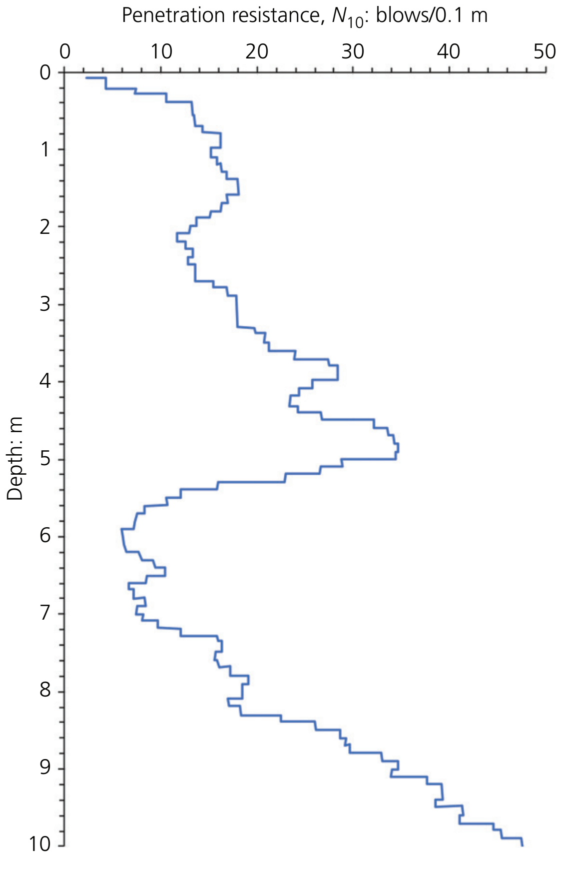

Figure 6. Average of 15 light dynamic probing tests (Schönit, 2009). The groundwater table was at 5.4 m depth

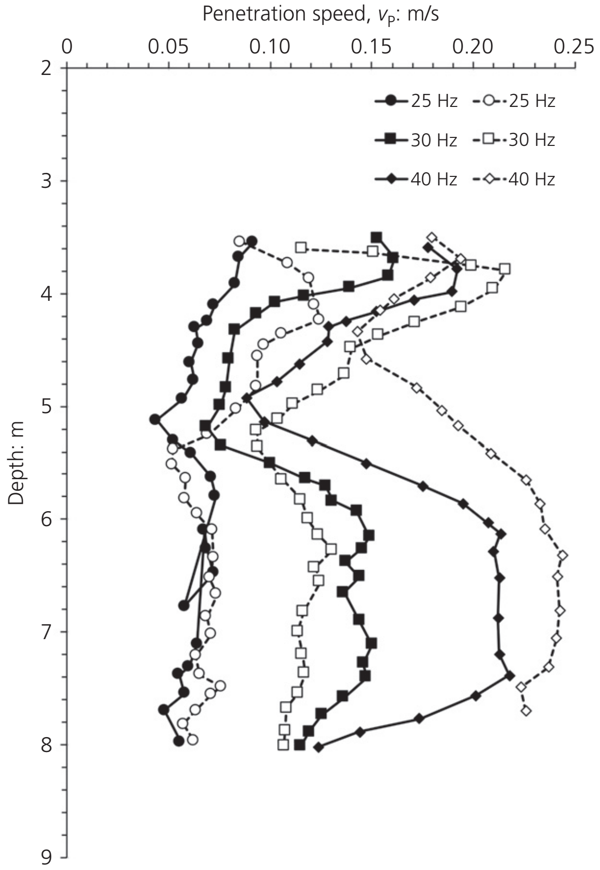

Figure 7. H-beam penetration speed, v_P, measured at three different operating vibration frequencies with two tests for each frequency (Schönit, 2009)

7. Resonance effects

Vibration frequency is an important parameter that influences all aspects of vibratory driving (driveability, bearing capacity and environmental effects). Resonance occurs when the frequency of a periodically applied force is equal or close to the system resonance frequency. When an oscillating force is applied at resonant frequency, the system will oscillate at an amplified displacement amplitude compared with when the same force is applied at a higher, non-resonant, frequency. The proposed theoretical models of vibro-driveability have generally not included the interaction of the vibratory-driven pile with the surrounding soil deposit. For instance, Masoumi and Degrande (2008) calculated free-field vibrations by means of a coupled finite-element–boundary-element model using a subdomain formulation. However, they did not consider the effect of vibration frequency on the ground response as their analysis investigated the case of only one vibration frequency (20 Hz).

The most reliable concept of determining system resonance is by field tests, which can be readily performed during vibratory driving or soil compaction (Massarsch and Wersäll, 2019). An example of a resonance field test is shown in Figure 8. In this test, a 300 mm dia., 10 m long, closed-toe pipe pile was vibrated into medium-dense sand. The pile was driven by a vibrator (MS 24HF VAR) with variable frequency (0–37 Hz) and an eccentric moment of 24 kg.m. The vibration response of the ground was measured during 3 min of driving, when the pile had penetrated to 6 m depth. The vertical ground vibration velocity was measured by a geophone located at a lateral distance of 4 m. In order to determine the system resonance, the vibrator frequency was varied between 11 Hz and 37 Hz and the ground vibration velocity in the vertical direction was recorded. It is apparent from Figure 8 that the vertical vibration velocity was strongly affected by the vibration frequency.

The results from Figure 8 were evaluated to demonstrate the effect of frequency on vertical ground vibrations. Figure 9 show the vertical ground vibration velocity as a function of vibration frequency. A distinct peak in vibration velocity can be observed around 14 Hz (system resonance frequency), where the vertical ground vibrations were strongly amplified (up to 15 mm/s). When the vibration frequency was increased above 30 Hz, the ground vibrations decreased (<3 mm/s). System resonance thus has two important consequences for vibratory driving – ground vibrations are strongly amplified (by a factor of five) and the probe penetration speed decreases significantly.

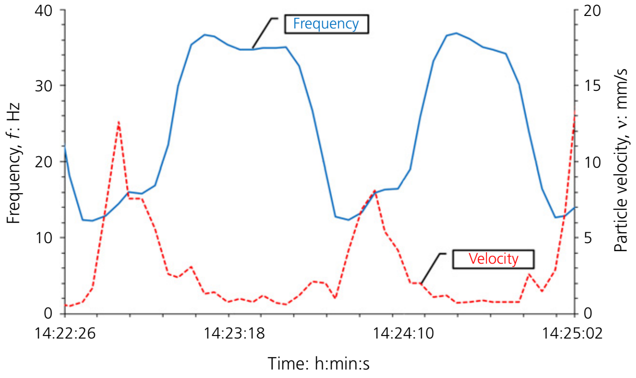

Figure 8. Vertical vibration velocity at 4 m distance during driving of a closed-toe pipe pile at 6 m depth during driving into sandy soil (data from Massarsch (2000))

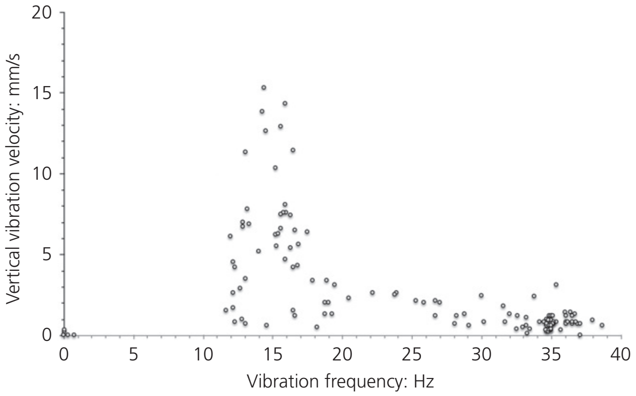

Figure 9. Frequency response of ground during vibratory driving of closed-toe steel pipe pile into sand (data from Figure 8, after Massarsch (2000))

It should be noted that the measured resonance frequency is not a soil parameter but depends on several factors, such as the vibrator mass, the pile mass, the pile penetration depth and the stiffness (shear wave speed) of the soil.

8. Pile bearing capacity

The general opinion of practitioners is that the bearing capacity of vibrated piles is lower than that of impact-driven piles (Bosscher *et al.*, 1998; Briaud *et al.*, 1990; Fischer *et al.*, 2013; Mazurkiewicz, 1986; O'Neill, *et al.*, 1990; Remspecher, 2014; Schönit, 2009). However, the effect of vibration frequency during pile installation on the bearing capacity has generally not been studied and evaluated. In addition, the consequences of the installation procedure during the final phase of pile driving (seating of the pile) are generally neglected. Two studies (Briaud *et al.*, 1990; Hartung, 1994) shed new light on these issues, as now discussed.

8.1 Model tests

Hartung (1994) performed 1g model tests to investigate the effect of driving frequency on the toe and shaft resistance of piles vibrated into sand. This thesis was published in German only and, regrettably, the results were never published elsewhere. The original data of Hartung (1994) were digitised and now hopefully presented in a legible format. The model tests were performed in a concrete cylinder of diameter 0.45 m and height 2 m. A 1.85 m thick sand fill was placed and compacted under controlled conditions and then water-saturated. The grain size characteristics of the sand were $d_{10} = 0.25$ mm and $d_{50} = 0.55$ mm with $C_u < 3$. Sand compaction was achieved with a vibrator that excited the concrete cylinder.

The test piles were straight-shafted steel pipes (36 mm outer diameter, 10 mm wall thickness and 1500 mm length). The pile toe was closed and flat. The steel piles were installed with a vibrator having a centrifugal force of 2000 N. Two test series were performed in which the vibrator mass to pile mass ratio was 1 : 1 or 2 : 1. The vibration frequency was varied in the range 20–50 Hz. The installation time for all piles was measured during the final 500 mm of penetration. The vibration amplitude was measured by means of accelerometers installed on the head and toe of the pile. In addition, an accelerometer was placed on the surface at a lateral distance of 0.18 m (half a pile diameter away from the pile). Acceleration measurements were converted by integration to vibration velocity. For all tests, frequency analyses were performed. An interesting aspect of this work was the measurement of pile temperature by sensors installed at the toe of the pile.

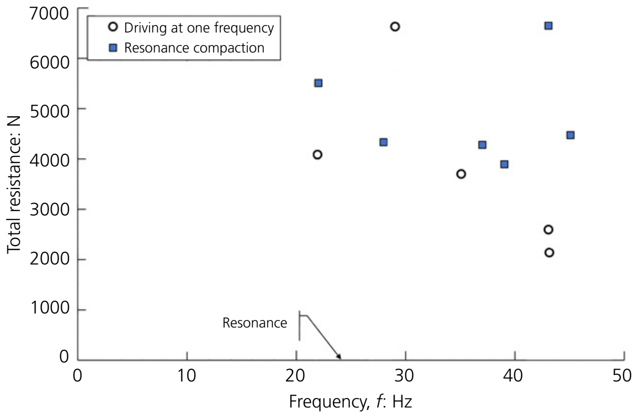

Figure 10. Total static pile resistance of piles installed at different vibration frequencies. Selected piles were subjected to additional vibration at resonance frequency (22 Hz) for a duration of 120 s (data from Hartung (1994))

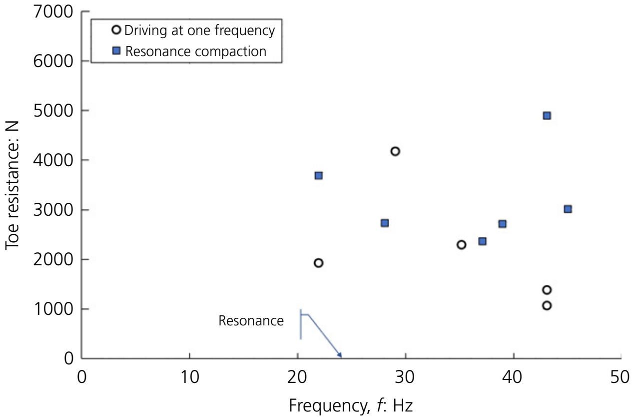

Figure 11. Toe resistance of piles installed at different vibration frequencies. Selected piles were subjected to additional vibration at resonance frequency (22 Hz) for a duration of 120 s (data from Hartung (1994))

The objective of this was to investigate whether a rise in temperature could be observed during vibratory driving at different frequencies. Such measurements could provide insight into the energy consumption (transfer of vibration energy into heat) during driving.

Static loading tests were carried out after the installation of each pile to determine the total compressive pile resistance and the pull-out resistance; the difference was considered to be the toe resistance. For all tests, the bearing capacity was defined to be the applied load that generated a 1.0 mm pile-head movement. Initial tests showed that the system resonance frequency was around 22 Hz. The test scheme was to first install the pile at one chosen frequency, between 20 Hz and 43 Hz. Some test piles were thereafter subjected to additional vibratory treatment at the resonance frequency (22 Hz) lasting for 120 s. The total pile resistance obtained from different tests is summarised in Figure 10. For all the piles installed at one frequency, the total pile resistance increased with decreasing frequency. However, the total pile resistance increased by a factor of about two to four when the pile was vibrated after installation at the resonance frequency (22 Hz) for a duration of 120 s. This effect was particularly pronounced for the piles installed at the highest frequencies (>30 Hz).

8.1.1 Toe resistance

Figure 11 shows the effect of driving frequency on the toe resistance evaluated as the difference between pull and push tests at 1 mm toe movement, for piles installed at high frequency and for piles with a post-installation treatment at 22 Hz (resonance frequency). The toe resistance of piles installed at a high frequency (>30 Hz) was significantly smaller – by a factor of three to five – than that of piles vibrated after installation at the resonance frequency. This difference is significant and shows that finishing driving with vibration at resonance frequency after installation at a high frequency will compact the potentially loosened soil at the pile toe and thus improve the stiffness of the pile toe response and bearing capacity of the pile.

8.1.2 Shaft resistance

Figure 12 shows the effect of driving frequency on the shaft resistance of the piles. The shaft resistance of piles installed at a high frequency (>30 Hz) was generally lower than that of piles installed at lower frequency. All piles – independent of the installation frequency – reached about the same bearing capacity (1500–1900 N) when vibrated at the resonance frequency of 22 Hz.

Inspection of the test results reported by Hartung (1994) indicates that vibrating a pile at – or close to – resonance frequency has two effects: (*a*) compaction of the soil, thereby increasing the bearing capacity and (*b*) a tendency to equalise the bearing capacity of piles installed at different frequencies. It could be argued that vibrating piles at resonance is creating a more uniform soil state. This observation agrees with anecdotal experience from resonance compaction projects (Li *et al.*, 2018; Massarsch and Fellenius, 2017). It should be noted that the model tests were carried out at 1***g*** so the effects of dilation in compacted sand could be very high and thus the effect of post-installation vibration might not be as significant at field scale (i.e. density changes due to initial driving might be smaller at full scale).

8.2 Field tests

The installation procedure can be of major importance for the bearing capacity of vibratory-driven piles, as shown in a case history reporting the installation of three H-piles (360HP108) impact-driven in a medium-dense sand deposit built up of a hydraulic fill, load tested in compression, extracted and reinstalled by vibratory driving 10 m away

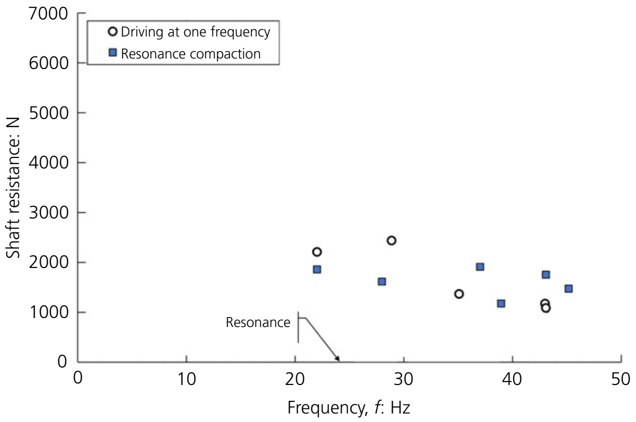

Figure 12. Shaft resistance of piles installed at different vibration frequencies. Selected piles were subjected to additional vibration at resonance frequency (22 Hz) for a duration of 120 s (data from Hartung (1994))

(Briaud *et al.*, 1990). Static loading tests were then also carried out on three vibratory-driven piles.

A Delmag diesel hammer (model D22) was used for impact driving. The vibratory hammer (ICE-216) had an eccentric moment of 11.5 kg.m, a displacement amplitude of 19 mm and a frequency of 6–26 Hz. The piles encountered very easy driving and the time required to drive the piles was the same for the impact and vibratory hammers. The vibratory installation method (e.g. vibration frequency, type of piling rig and clamp mass) was not reported in the paper, which is, regrettably, typical for many case histories. The static loading tests showed that, compared with impact driving, vibratory driving of piles resulted in (*a*) approximately the same maximum resistance at large movements, (*b*) a large scatter in this maximum load from one pile to the next and (*c*) a larger movement at working (unfactored) loads. The toe resistance of impact-driven piles was 51% of the total resistance, but only 13% for the vibratory-driven piles.

The static tests involved reloading the piles. The reloading load–movement curves took on the same shape for the impact-driven piles. However, for the vibratory-driven piles, the toe resistance was larger. As the soil conditions were relatively homogeneous, it was concluded that the variability in toe resistance was due to operator-related factors. It is also likely that the effect of the first test left the pile with a stiffened toe response. The maximum displacement amplitude of the vibrator was 19 mm. Although the displacement amplitude of the pile was not measured, it can be assumed to correspond to at least 10 mm (Equation 3). Thus, at the end of vibratory driving, the pile was still oscillating and not fully in contact with the soil below the pile toe, which can explain the low toe resistance in the first test (cf. Figure 3). In addition, unintentional holding back or even lifting up of the vibrator and pile by the machine operator during the final phase of driving could have reduced the toe resistance. Thus, it is important to appreciate that vibratory driving is more strongly operator-dependent than impact driving.

Many challenges remain when estimating the toe resistance of vibratory-driven piles and sheet piles. One unresolved issue is the influence of the toe geometry, which differs between sheet piles and open-toe and closed-toe pipe piles. For instance, the formation of a soil plug in a pipe pile is still an unresolved issue. The vibration frequency (resonance effects) can influence plug formation at the toe of a pipe pile.

9. Interaction of vibrating pile and soil

Various concepts have been proposed to explain the mechanism of vibratory driving and its effect on soil surrounding a pile. In spite of detailed analytical studies and field measurements, the interaction of the vibrator–pile–soil system has not yet been explained satisfactorily. It is generally accepted that when a pile is vibrated at a high frequency (>30 Hz), part of the energy will be transferred into heat and sound. An occasionally mentioned factor, supposed to reduce the pile shaft friction, is liquefaction. However, this effect is not relevant as it can only occur in loose, water-saturated granular soils. Therefore, other factors need to be considered to explain the efficiency of vibratory driving in granular soils.

9.1 Friction fatigue

The term ‘friction fatigue’, initially introduced by Heerema (1980) for impact-driven piles, was used to illustrate the effect of horizontal stress changes on pile driveability. Experiments with instrumented displacement piles had shown that the ultimate shaft friction that can develop in a given sand horizon decreases as the pile tip penetrates to deeper levels. This phenomenon is now commonly referred to as friction fatigue. White and Lehane (2004) investigated friction fatigue in centrifuge tests. The soil consisted of fine silica sand and model piles equipped with lateral stress sensors were installed by three different methods (monotonic, jacking and pseudo-dynamic). However, none of the model piles were installed by vibratory driving. It was found that the primary mechanism controlling friction fatigue was the cyclic history imparted during pile installation to soil elements at the pile–sand interface. For a given installation method, the stationary lateral stress acting at any given level on a displacement pile can be described as a relatively unique function of the CPT cone penetration resistance and the number of cycles imposed during installation. The strong influence of cycling, which is also seen in cyclic, constant normal stiffness interface shear tests, can be attributed to contraction of a narrow shear zone at the shaft–soil interface that is surrounded by soil with a relatively high lateral stiffness.

Moriyasu *et al.* (2018) studied friction fatigue during vibratory driving. They determined the accumulated shear work during vibratory driving, which is the product of the number of cycles, the amplitude of displacements at the pile head and the vibratory forces. However, no generally applicable concept was offered that could explain the vibratory driving process in different soils. Their field measurements showed that, even in loose silty sand, the excess pore water pressures generated during vibratory driving were low and even negligible in terms of interpreting the behaviour of the piles.

9.2 Arching around pile shaft

In order to study the effect of vibratory driving on surrounding soil, vibration measurements have been performed below the ground surface during vibratory driving (Deckner, 2017; Krogh and Lindgren, 1997; Massarsch, 2002). These measurements suggested that when penetrating granular soil, a vertically vibrating pile generates a horizontally oscillating stress field with vibrations directed away from and towards the pile shaft (see Figure 4). Pile driving in clay, however, does not seem to cause noticeable horizontal vibrations (Deckner, 2017). The horizontally oscillating stresses create a densified soil cylinder around the pile, through which the larger horizontal stress away from the pile will arch. Therefore, the effective stress against the pile surface during vibratory driving at high frequency will be lower than before the start of driving. This concept holds for dry and partially or fully water-saturated soils. Moriyasu *et al.* (2018) measured horizontal stresses against the pile shaft during vibratory driving of steel pipe piles into silty sand: a marked decrease in shaft friction was observed after vibratory driving.

Axelsson (2002) observed horizontal stress changes against the pile shaft as a result of impact driving a pile into sand. In this study, a single 20 m long concrete pile instrumented with earth stress cells at five different depths along the pile shaft was driven into well-graded silty sand to gravelly sand. The groundwater table was located approximately 2 m below the ground surface. The density index, I_D, was 35–50% and the friction angle, ϕ, was approximately 33°. The coefficient of lateral earth stress, K_0, before driving was estimated to 0.45. The horizontal effective stresses measured against the pile surface at different times after driving are shown in Figure 13. Immediately after driving, the measured horizontal stress was significantly lower than the original horizontal effective stress. It increased with time after driving, but after 70 days it was still less than half the original value.

Axelsson (2002) concluded that the impact driving of the single pile in sand generated a densified soil cylinder, causing arching around the shaft and thereby reducing the horizontal stress acting against the pile shaft. Arching is associated with high tangential and low radial compressive stresses. However, over time, the horizontal stress against the shaft increases due to the gradual collapse of arching around the shaft (Figure 13). Axelsson (2000) reported that the horizontal stresses increased over a period of 70 days and were approximately linear with the logarithm of time. Furthermore, the degree of stress relaxation was observed to increase strongly with depth. Resonance compaction of piles driven at high frequency can contribute to the collapse of horizontal stresses and a simultaneous densification of the zone adjacent to the pile shaft. This concept was confirmed by the model tests reported by Hartung (1994), which showed that the shaft bearing capacity of piles vibrated into sand at high frequency (>30 Hz) was low compared with that of a pile driven at the resonance frequency (22 Hz). It should also be noted that the driving of additional piles, or re-driving of the pile at a later time, could further enhance the collapse of the arching cylinder and thus increase horizontal stresses acting along the pile shaft.

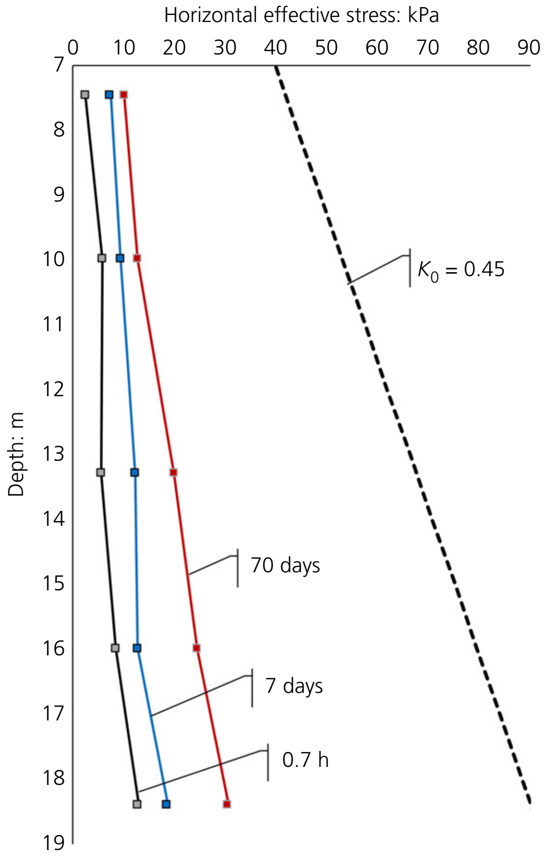

Figure 13. Horizontal effective stress acting at shaft of a single pile at different times after impact driving (data from Axelsson (2000)). Also shown is the estimated horizontal effective stress before pile driving, assuming $K_0 = 0.45$

Lehane and White (2005) described a similar series of tests performed in a drum centrifuge. Instrumented model displacement piles were driven into normally consolidated sand. They found that the cycling loading associated with pile installation resulted in a progressive reduction in the stationary horizontal effective stress acting on the pile shaft and densification of the sand in a shear band outside the pile shaft. These findings agree with the observations of Moriyasu *et al.* (2018) and Axelsson (2000).

Based on the horizontal stress measurements below the ground surface reported by Krogh and Lindgren (1997), it can be concluded that horizontal vibration cycles are created during vibratory driving of piles at high frequency. These vibrations reduce the effective stress temporarily during driving and contribute to the arching horizontal stresses around the shaft of the pile.

Jardine *et al.* (2006) reported the results of tension tests on impact-driven steel pipe piles in dense sand. The tests showed a marked increase in shaft capacity over time. The aged piles exhibited surprisingly brittle failure modes – prior testing to failure both degraded the capacity and modified the ageing processes. Jardine *et al.* (2006) noted that gentle vibration accelerates creep in granular media, supporting the hypothesis that the observed ageing and pre-testing trends originate from a circumferential arching action that (*a*) becomes more marked after each extreme load cycle (involving slip) associated with driving or testing and (*b*) weakens over time due to creep. These field data are consistent with the previously offered explanation for the time dependence of the shaft capacity of piles driven in sands: that the radial stresses developed on the shaft increase through the relaxation (over time) of a circumferential arching stress field.

10. Proposed concept to explain the vibratory driving process

Based on the information presented thus far, substantiated by comprehensive investigations by different researchers, it is possible to propose a hypothesis to describe the efficiency of vibratory pile driving in granular soil.

When a pile is driven at a high vibration frequency (typically >30 Hz), horizontally oscillating vibration cycles are generated that temporarily reduce the shearing resistance along the pile shaft. In addition, during pile penetration, the soil will be disturbed in a narrow zone surrounding the pile shaft. The effect of the interaction of the vertically oscillating pile and the soil at high frequency (>1.5 times the system resonance frequency) is illustrated in Figure 14(a). Soil disturbance (densification or loosening) will occur in a narrow zone surrounding the pile shaft. At a high vibration frequency, ground vibrations will be low and attenuate rapidly with increasing distance. According to the above proposed concept, an arching zone will be created around the vibrated pile, resulting in a reduction in the horizontal effective stress acting at the pile shaft. During high-frequency vibratory driving, most of the vibration energy will be emitted from the toe of the pile. When the vibrator is operated at – or close to – the system resonance frequency, the pile can efficiently transmit the vibration energy to the surrounding soil. Strong ground vibrations will occur, resulting in soil densification. The compacted zone will gradually extend to a larger distance from the pile when compared with high-frequency driving. During resonance compaction, the arching zone surrounding the pile shaft is likely to collapse (Figure 14(b)). This assumption is supported by the investigations reported by Hartung (1994).

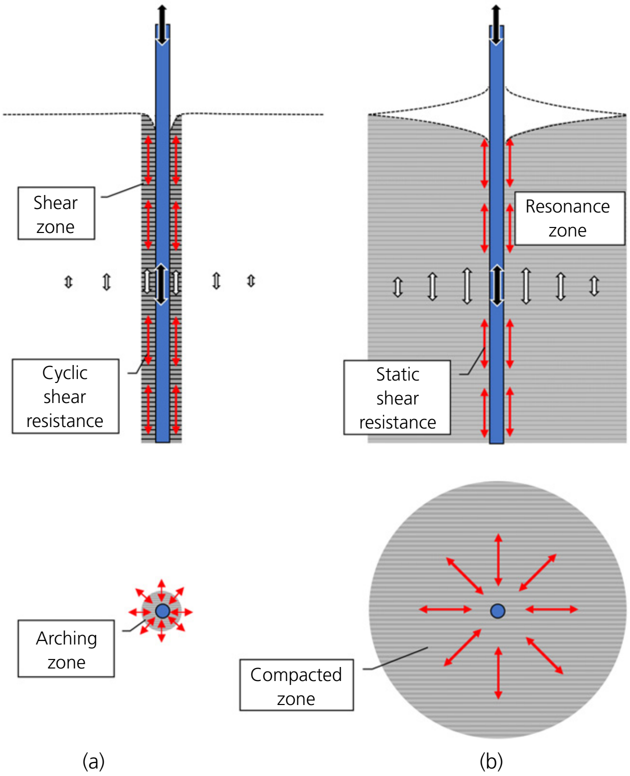

Figure 14. Conceptual sketch of the effect of pile–soil interaction during vibratory driving: (a) driving above the system resonance frequency; (b) driving at system resonance frequency

According to this concept, the most efficient method for vibratory driving of piles into granular soil is to start driving at a high frequency, thereby achieving a high penetration rate and low emission of ground vibrations. When the required penetration depth is reached, the vibration frequency should be lowered to approach system resonance, which causes soil densification. As a result of gradual compaction, the arching zone around the pile shaft will collapse, thereby increasing the horizontal effective stresses against the pile shaft and stiffening the pile response. It should be emphasised that the proposed mechanism is still not fully understood or proven and further investigations are needed to substantiate this concept.

11. Conclusions

Ever more powerful and sophisticated vibrators are becoming available, for which different operating parameters (e.g. frequency and eccentric moment) can be used to optimise the driving process, increase pile bearing capacity and minimise environmental impacts. An important but generally not appreciated fact is that in contrast to impact pile driving, the execution of vibratory driving is strongly dependent on the operator. This aspect is of particular importance for the bearing capacity of piles during the final phase of pile driving (seating of the pile). During impact driving, the pile toe is always in contact with the underlying soil. However, this is not the case during vibratory driving, in which the pile toe is oscillating. Thus, when the vibrator is shut down, the pile toe can be separated from the foundation layer. In addition, as the vibrator is rigidly connected to the pile, the machine operator can (often unintentionally) pull the vibrator and thus the pile, thereby reducing the contact of the pile toe with the foundation layer.

A fundamental difference between impact driving and vibratory driving is that, during vibratory driving, resonance effects can either be avoided (when advancing the pile at high frequency) or used to increase soil compaction. Driving a pile at the system resonance frequency will increase ground vibrations and reduce the pile penetration speed. Relatively simple field measurements can be used to determine the system resonance frequency in order to avoid or seek system resonance as desired. For efficient driving, it is recommended that the operating frequency of a vibrator is at least 1.5 times higher than the system resonance frequency.

Vibration measurements below the ground surface have shown that in granular soils – where vibratory driving is most effective – the vertically vibrating pile gives rise to horizontally oscillating stresses. These stress changes temporarily reduce the normal stresses acting against the pile shaft, thus explaining the effectiveness of vibratory driving in granular soils.

An important question is the bearing capacity of vibrated piles, which is frequently lower than that of impact-driven piles. Model tests have shown the significance of vibration frequency for the bearing capacity of piles in sand. A pile driven at high frequency achieves high penetration rates and low environmental impact but, generally, a lower bearing capacity. However, if a pile is vibrated during the seating phase at the system resonance frequency, significantly higher bearing capacity can be achieved.

Measurements of horizontal stresses acting against the shaft of piles driven or vibrated into sand show low normal stresses. This effect can be explained by the formation of a densified soil cylinder, causing arching of the horizontal stresses, which results in the low shaft resistance. Over time, this arching effect will decrease, especially if piles are driven in the vicinity.

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